

LARGE DISPLACEMENT EFFECTS ON SEISMICALLY EXCITED ELASTIC-PLASTIC FRAME STRUCTURES

C. Adam*

Department of Civil Engineering, Vienna University of Technology
Wiedner Hauptstrasse 8-10/E201, A-1040 Vienna, Austria
and

H. Krawinkler

Department of Civil and Environmental Engineering, Stanford University
Terman Engineering Center, 380 Panama Mall, Stanford, CA 94305-4020, USA.

ABSTRACT

Recently developed nonlinear beam/columns elements based on large displacement theory are easily accessible to the research community for the dynamic analysis of earthquake excited structures. It is of interest to assess whether the application of these elements is desirable in engineering practice and research. This study is intended to provide quantitative knowledge on the importance of large displacement effects on the response of seismically excited structures. Generic frame models are utilized to represent SDOF and MDOF structures. Equivalent pulses are used to represent seismic input, since their effect on structural response is comparable to that of near fault ground motions. Large displacements of frame structures give rise to second-order amplification and thus, structure and member P-delta effects are addressed. The results demonstrate that the influence of large displacement formulations is of secondary importance for the response prediction of elastic-plastic SDOF and MDOF frame structures.

Keywords: large displacements; generic frame structures; pulse-type ground motion; nonlinear response; P-delta effect; seismic response

1. INTRODUCTION

With increasing computer processing power the methods of structural analysis have emerged from linear static, linear dynamic, nonlinear static to nonlinear dynamic analysis. Until now most dynamic analyses in earthquake engineering are based on small displacement theory, i.e. axial strains and angles of rotations in beam/column elements are linearized, and nonlinearity is confined to inelastic material behavior. In recent years the object oriented finite element code OpenSees (<http://opensees.berkeley.edu/>) is developed

* E-mail address of the corresponding author: ca@allmech.tuwien.ac.at

within Pacific Earthquake Engineering Research (PEER) center (<http://peer.berkeley.edu>) in an effort to provide a powerful tool for the dynamic analysis of structures exposed to seismic excitation. The element library of OpenSees includes two beam/column elements, which are based on large displacement theory, and they are easily accessible to the research community. These elements are an "Updated Lagrangian" beam/column element, which takes the member P-delta effect into account, and a "Corotational" beam/column element. While for elastic structures the validity of small displacement theory is well known, for structures driven into the inelastic range of deformations the perceived need to transition from small to large displacement theory is not well determined yet.

This study attempts a systematic evaluation of large displacement effects in the prediction of the elastic-plastic dynamic response of structures exposed to seismic excitation. The global objective is to acquire quantitative knowledge on large displacement effects, and achieving a fundamental understanding of structural response characteristics with respect to large displacements, see also Refs [1 - 4]. Structures are described by generic elastic-plastic single- (SDOF) and multi-degree-of-freedom (MDOF) frame systems. In this paper results referred to as *large displacement theory* are derived by utilizing the "Updated Lagrangian" beam column element of the OpenSees finite element code (<http://opensees.berkeley.edu/>). Since large displacements of frame structures give rise to second-order amplification, the structure and member P-delta effect are addressed in this study. Some evidence of the effects of P-delta on the collapse of simple frames is available from a recent experimental study by Bruneau and Vian [5].

Pushover analyses are performed, and displacements predicted by application of small and large displacement theory are contrasted. Subsequently, generic frame structures are exposed to pulse type ground excitation, which represent in a simplified manner near-fault ground motion, see Ref. [6]. Near-fault ground motions tend to impose large displacement demands and hence, large displacement theory might affect the prediction of the dynamic response. Peak displacement (drift) spectra and drift demands for specific structural periods are investigated through a comprehensive parametric study that describes the variation of seismic demands with structure parameters such as fundamental period and base shear strength. Very (for technical applications unrealistic) large structural deformations with ductilities up to 50 are considered in an effort to illustrate the effect of large displacements over a wide range.

2. LARGE DISPLACEMENT RESPONSE OF SDOF SYSTEMS

2.1 Generic SDOF model

A one-story portal frame composed of two rigid columns, a rigid beam, two point masses m_s and two rotational springs of elastic stiffness K_{sp} as shown in Figure 1 is utilized in the subsequent studies of SDOF systems. Inelastic structural behavior is defined by a nondegrading elastic-perfectly plastic moment-rotation hysteresis rule of the springs. No strain hardening is taken into account. It is expected that dynamic P-delta effects will be of major concern for structures exposed to large displacement pulses

and hence, identical gravity loads are assigned to each corner. All numerical simulations are performed without consideration of viscous damping. Viscous damping of dynamically excited elastic-plastic structures leads to problems in formulation of static equilibrium. This topic is not addressed in this investigation. A detailed discussion of viscously damped elastic-plastic structures can be found in Medina [7].

Fundamental studies are carried out with elastic-plastic SDOF systems in order to capture basic response characteristics that differentiate large from small displacement theory. The initial linear period T of the SDOF system is varied at closely spaced intervals to provide accurate spectral information within the range of interest. As T varies, the properties of the springs (yield rotation θ_y , and/or yield moment M_y [or yield strength $F_y = 2 M_y / h$]) should be varied in a manner that keeps the structural properties within a practical range. In this study it is assumed that the yield rotation of the springs, θ_y , stays constant while T is varied, and the yield strength F_y is adjusted accordingly. From a structural point of view this is a reasonable assumption, particularly for steel moment-resisting frames [8].

Yielding of a SDOF system is quantified by the yield strength coefficient γ , which is defined as the ratio of yield strength F_y over structural weight W . For the considered SDOF systems γ is a function of the linear structural period T , the structural height h , and the plastic yield rotation θ_y . From $K_e = F_y / (\theta_y h) = 4\pi^2 m / T^2$ it follows:

$$\gamma = \frac{F_y}{mg} = \frac{4\pi^2}{T^2} \frac{h}{g} \theta_y, \quad m = 2m_s, \quad (1)$$

where g denotes the acceleration of gravity. Since the yield rotation θ_y is a constant in the entire range of considered structural periods T , the height h is the only remaining parameter to be assigned. It is desirable that the definition of h leads to rational values of the stability coefficient $\theta_s = P / (h K_e)$ which characterizes P-delta effects on the elastic structural response. For the considered SDOF system it is assumed that h is a function of its initial elastic natural period T , i.e., $h = \beta T$. The coefficient β is tuned to obtain a stability coefficient corresponding to the first story stability coefficient $\theta_{s,1}$ of an elastic MDOF multi-story frame.

The stability coefficient θ_s of the elastic SDOF system of Figure 1, defined as the ratio of geometric over elastic structural stiffness K_e , can be derived as

$$\theta_s = \frac{T^2}{4\pi^2} \frac{\alpha g}{h} \quad (2)$$

Figure 2. Yield strength coefficient as a function of the structural period

2.2 Static pushover analysis

At first, the effect of large displacements is evaluated by means of a static pushover analysis. Thereby, a horizontal force H is applied incrementally at the beam level and the corresponding story drift Δ_h / h is computed. Figure 3 shows normalized base shear versus story drift diagrams. Results derived by large and small displacement formulations are set in contrast. For the examples shown, P-delta effects are characterized by a stability coefficient θ_s of 0.1. Results are presented for yield rotations $\theta_y = 0.01, 0.05$, and 0.1 . It is recognized that the latter two values are irrelevant for practical applications, but it is necessary to employ such large values in order to illustrate any effect of large displacement theory.

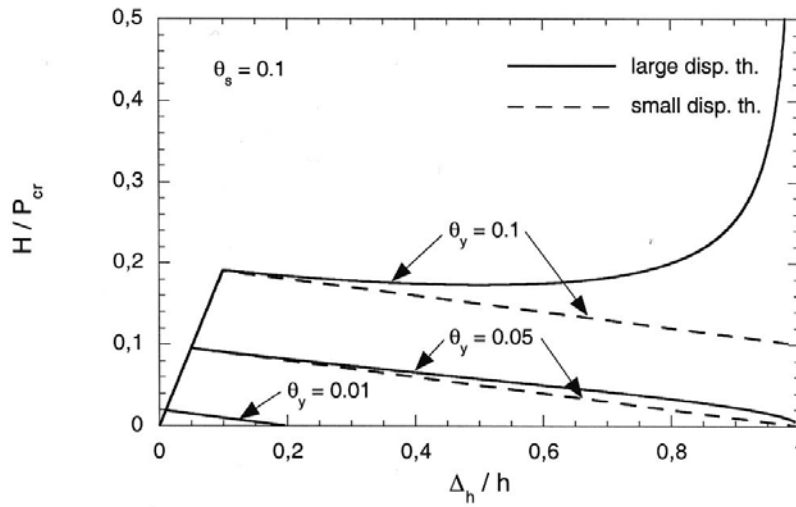


Figure 3. Pushover curves for various yield rotations, P-delta effects considered, large versus small displacement theory

The outcomes of this study become independent of geometry and elastic material parameters by normalizing the base shear H with respect to the linear buckling load $P_{cr} = K_{sp} / h$. If large displacement theory is applied the maximum story drift Δ_h / h at collapse is 1. In all of the graphs of Figure 3 beginning of structural yielding is indicated by a sharp kink. According to this figure gravity loads lead in the small displacement formulation to a negative post-yield stiffness under all conditions. Small displacement theory provides an excellent estimate of the story drift up to $\Delta_h / h = 0.2$. Utilizing large displacement theory the normalized base shear goes to infinity for a yield rotation of $\theta_y = 0.1$, whereas for $\theta_y = 0.05$ displacements obtained from both theories are close for the entire range of the story drift. A practical value of the yield rotation, $\theta_y = 0.01$, together with a stability coefficient of $\theta_s = 0.1$ leads to structural collapse before the effect of large displacements becomes visible.

2.3 Response of SDOF systems to seismic pulse input

Fundamental studies are carried out with SDOF systems exposed to pulse-type input in order to capture dynamic response characteristics that differ between large and small displacement theories. As shown in [6] the basic characteristics of near-fault ground motion can be represented by relatively simple pulses. For this particular study a pulse of type P2 has been adopted [6]. Thereby, the foundation of the structure experiences a reversing displacement history that is generated through a double cycle of acceleration input. The pulse P2 acceleration history with pulse period T_p is represented by a square wave as shown in Figure 4, which results in a triangular velocity cycle and a second-order parabolic reversing displacement input. The intensity of the pulse is characterized by its peak ground acceleration a_p . Alavi and Krawinkler [6] have developed regressed relationships between pulse period and earthquake magnitude, and between peak pulse velocity and earthquake magnitude and distance to rupture zone, R . Using these relationships together with the equation $v_{p,max} = a_p(T_p / 4)$, representative values for a_p for magnitudes 6.0, 6.5, and 7.0 are 0.14g, 0.21g, and 0.31g, respectively, at $R = 3$ km, and 0.11g, 0.17g, and 0.25g, respectively, at $R = 5$ km. Representative pulse periods for magnitudes 6.0, 6.5, and 7.0 are 1.3, 1.8, and 2.6 s, respectively.

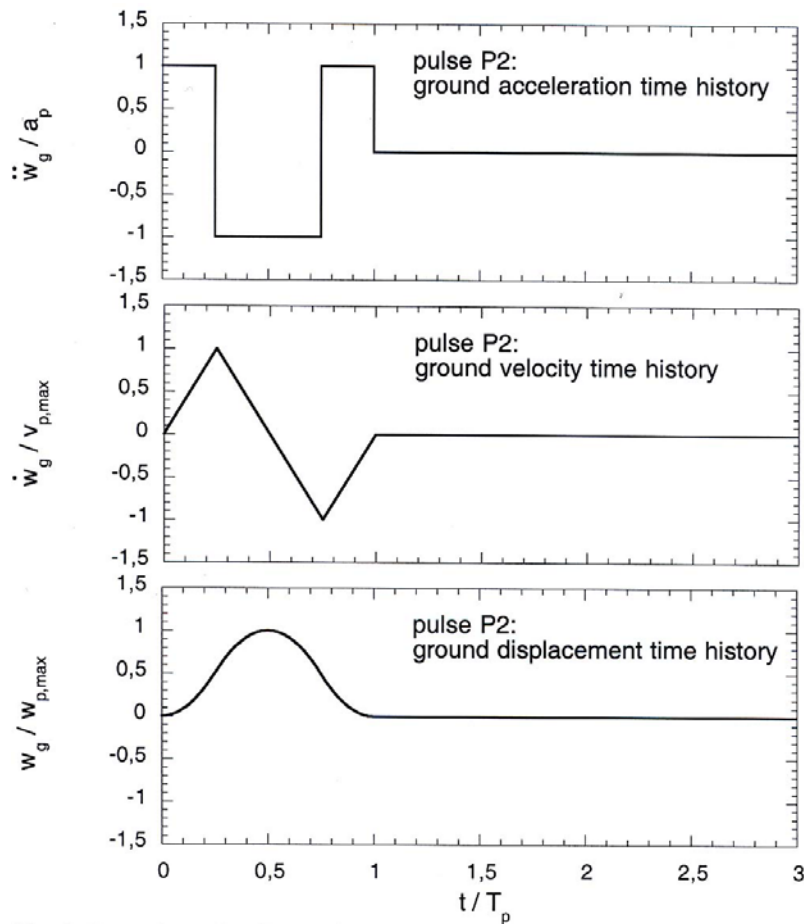


Figure 4. Ground acceleration, velocity and displacement time histories of pulse P2

For the results presented in Figs. 5 to 7 the system parameters previously summarized for Figure 2 are utilized, i.e., $h_1 = 3.66 \text{ m}$, $\varepsilon = 0.2$, and $\theta_y = 0.01$. The corresponding yield strength coefficient γ is shown in Figure 2 as a function of the period T . A coefficient of $\alpha = 1$ indicates that the full dead load (but no live load) is considered for structure P-delta effects, whereas $\alpha = 0$ identifies results neglecting structure P-delta.

Drift (Δ_h / h) response spectra for pulse P2 are presented in Figure 5 for selected values of pulse intensity a_p / g of 0.125, 0.25, 0.5, 1.0, 1.5 and 2.0 (the last three values, which are outside the practical range, are employed only to demonstrate the lack of importance of large displacement formulations). The structural period T is varied at closely spaced intervals, while the pulse period is selected to be $T_p = 2 \text{ s}$. Periods T , representing the initial structural period of the SDOF system without considering P-delta, are normalized by T_p . The spectral amplitudes are found from nonlinear time history analysis. Drift spectra derived with large and small displacement theory are set in contrast. The applied pulse intensities lead to very large drifts with ductility demands up to 44 (ductility demands can be seen by inspection since in all cases the yield drift is 0.01). The purpose of considering this unrealistic range of drifts is to illustrate the effect of large displacements over a wide range. Figure 5 illustrates that small displacement formulation of the equations of motion describes accurately the peak story drifts for the full range considered. Only for an (unrealistic) large pulse intensity of $a_p / g = 2$ a deviation of the results derived by both methods of analysis can be observed. It can be concluded that in the technical range of interest peak displacements can be predicted accurately by applying standard structural analysis programs (based on small displacement theory), which are readily available.

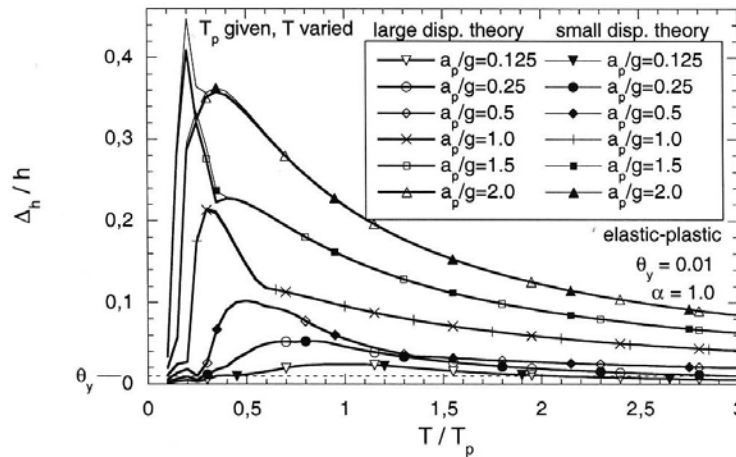


Figure 5. Drift response spectra for various pulse intensities a_p / g , pulse period $T_p = 2 \text{ s}$, structural period T varied

To emphasize this conclusion and to provide better insight into the response of SDOF

systems to pulse inputs, a different perspective of the results is presented in Figure 6. A relative strength parameter η , defined as $\eta = F_y / (m a_p) (= \gamma g / a_p)$, [6], is utilized to define the yield strength in relation to pulse severity. In Figure 6 results obtained from nonlinear time history analyses are presented as $\eta - \Delta_h / h$ (relative strength vs. drift) diagrams. The yield rotation is a constant in all numerical simulations: $\theta_y = 0.01$. Diagrams for selected ratios of $T / T_p = 0.25, 0.5, 1.0$, and 2.0 , all with a pulse period of $T_p = 2$ s are shown. It can be observed that with increasing pulse intensities (i.e. at low relative strength levels η) the drift demands become larger the smaller the period ratios T / T_p are. This effect can be observed also in Figure 5: the larger the pulse acceleration the more the peak response is shifted to small T / T_p values. Figure 6 illustrates that the small displacement formulation provides an accurate estimate of the displacement demands of these SDOF systems. Only for a short period structure with $T / T_p = 0.25$ at very large story drifts of $\Delta_h / h > 0.3$ (ductility > 30) a deviation of the response derived by both methods of analysis can be observed.



Figure 6. Relative strength parameter η versus drift for selected period ratios T / T_p

Normalized displacement response time histories for a SDOF portal frame exposed to pulse P2 are presented in Figure 7, using $T / T_p = 0.25$, $\theta_y = 0.01$, and $a_p / g = 2.0$. Again a pulse period T_p of 2 s is selected. The question is how small vs. large displacement formulation of analysis and gravity loads affect the time history of the response. The following observations can be made from this figure. The very large pulse intensity of $a_p / g = 2$ causes in an elastic-plastic system (with a yield drift of $\theta_y = 0.01$) a single large cycle with a positive and negative excursion and a large residual displacement. Most of the inelastic deformation occurs during the forced vibration phase between $0 < t < T_p$. The response including the P-delta effect ($\alpha = 1$) reaches its maximum during the first half of the pulse, whereas the analysis without gravity loads ($\alpha = 0$) provides the peak displacement at

the second reversal of the response. The absolute amount of this maximum is larger than that for the analysis with $\alpha = 1$. Gravity loads induce an additional overturning moment, which causes a larger maximum at the first reversal. However, when the frame oscillates in the other direction the overturning moments due to gravity loads and due to inertia forces work against each other, which finally leads to a smaller reversal peak displacement for $\alpha = 1$. This response behavior clarifies why in certain period ranges the P-delta effect reduces the drift compared to an analysis without gravity load. The reversal phenomenon is also responsible for the observation that drifts obtained from a small displacement formulation - if at all discernibly different - are usually larger than those obtained from a large displacement formulation.

As seen in Figure 7 the drift of the elastic-plastic system is very much larger than that of the elastic system because plastic deformation leads to structural period elongation, which brings the effective T/T_p ratio closer to unity.

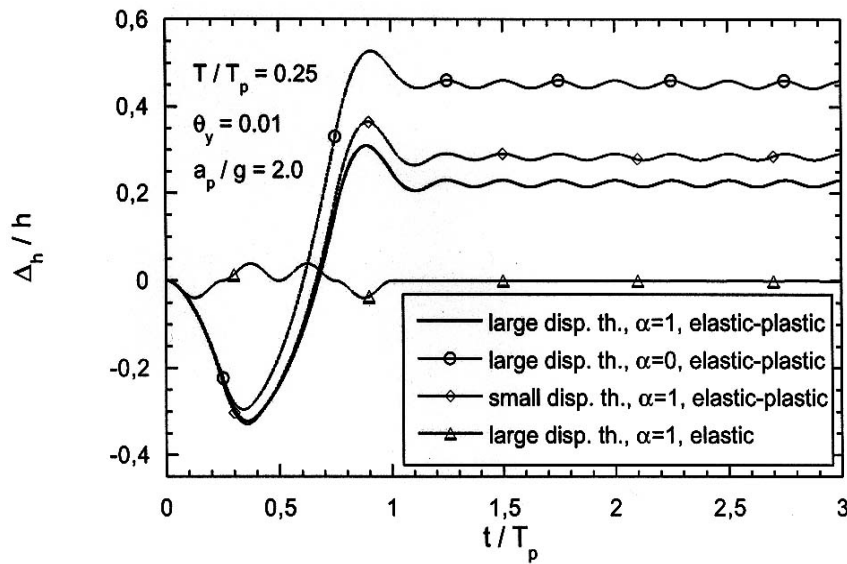


Figure 7. Drift time histories for a period ratio $T/T_p = 0.25$, pulse intensity $a_p/g = 2.0$

3. LARGE DISPLACEMENT RESPONSE OF MDOF SYSTEMS

3.1 Generic MDOF model

In order to quantify large displacement effects for MDOF structures a generic 18 story single-bay frame designed by Medina [7] is utilized. It is composed of rigid beams, elastic flexible columns and nondegrading elastic-perfectly plastic rotational springs, which are arranged at both ends of the beams, see Figure 8. No strain hardening in the post yielding range of deformation is taken into account.

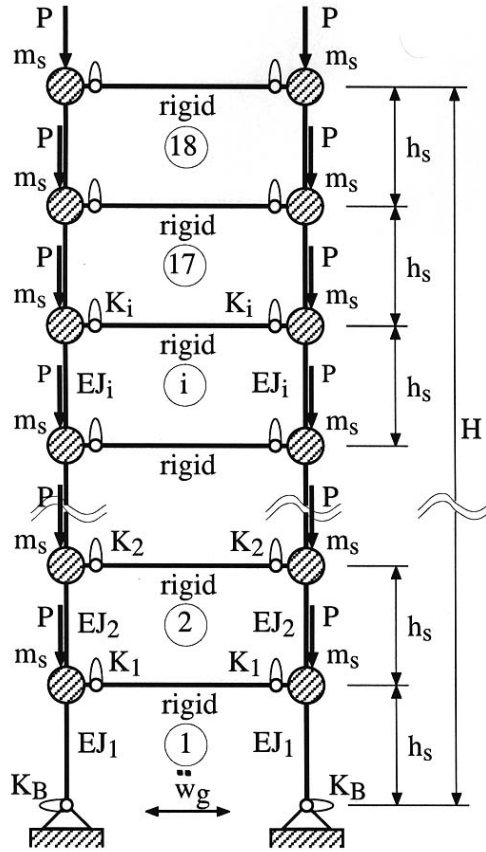


Figure 8. Idealized MDOF model of a 18-story single bay frame

The variation of the moment of inertia of columns over the height and the spring stiffness are tuned such that the first mode has a straight-line deflected shape. To each corner of the frame an identical point mass m_s is assigned. Given the base shear strength V_y , the strength of each spring is tuned such that all stories yield simultaneously in a static pushover analysis according to a lateral design load pattern that follows a parabolic distribution. For MDOF systems V_y is described by a base shear coefficient γ , defined as $\gamma = V_y / W$ [6], where W is the seismically effective weight. P-delta effects are simulated by assigning identical gravity loads to each story. This implies that axial column forces increase linearly from the top to the bottom of the frame. The magnitude of the gravity loads are related to the first story stability coefficient $\theta_{s,1}$ according to Eq. (3). For the frame used in this study the initial fundamental period T_1 (without considering P-delta) is selected to be 3.6 s, and the base shear coefficient γ is selected as 0.1. The story height is $h_s = 3.66$ m and the point mass applied at each node is $m_s = 45344$ kg. All numerical simulations are performed without consideration of viscous damping.

3.2 Response of MDOF systems to seismic pulse input

As in the study of SDOF systems, on the input side a pulse of type P2 [6] is adopted in an effort to represent near-fault effects on the structural response in a simplified manner. A relative strength coefficient η as defined in [6], i.e., $\eta = V_y / (M a_p)$ ($= \gamma g / a_p$) is employed to define base shear yield strength in relation to pulse intensity. M ($= 36 m_s$) denotes the total mass of the structure.

In Figure 9 results obtained from nonlinear time history analyses are presented in terms of relative strength coefficient η versus peak roof drift (Δ_H / H) diagrams. In the frame used in this study, gravity loads lead to a first story stability coefficient $\theta_{s,1}$ of 9.3 % (for $\alpha = 1.0$, i.e., no live load effects). Results are presented for $T_1 / T_p = 0.75, 1.0$ and 2.0 . In the analyses T_1 is kept constant (3.6 s.), while the pulse period T_p is varied. In the numerical simulations the pulse intensity a_p is stepwise increased. Figure 9 demonstrates that a decrease in η values (increase in pulse acceleration) leads to growing displacement demands before they stabilize and even become smaller. This phenomenon is caused by migration of the maximum story ductilities from the top portion of the structure to the bottom stories, [6]. With a further decrease in η the peak roof drift increases rapidly until structural collapse occurs. The smaller the T_1 / T_p ratio the larger the η at collapse.

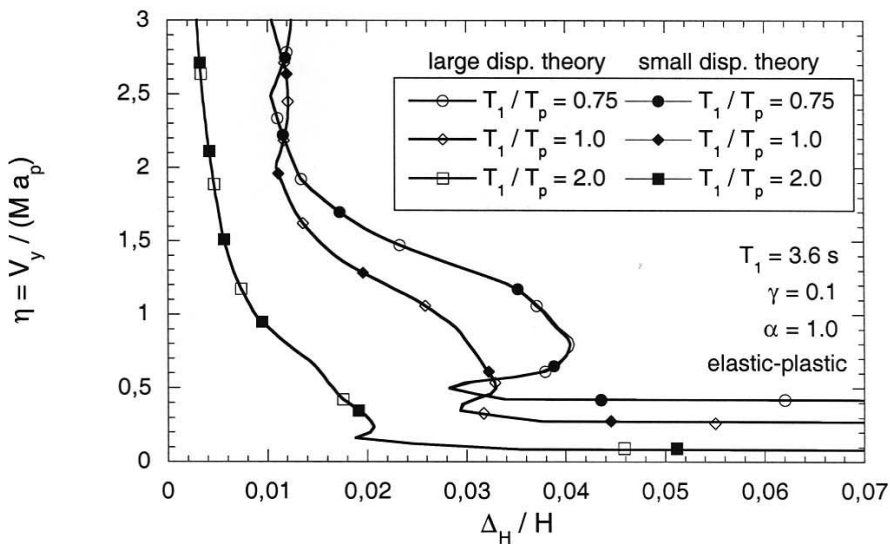


Figure 9. Relative strength parameter η versus roof drift for selected period ratios T_1 / T_p

The figure contrasts peak roof drift demands derived by large (heavy lines) and small displacement (light lines) theory. Since both theories give the same prediction of the peak response only heavy lines are visible. Figure 10 shows the corresponding maximum interstory drifts as a function of η . A comparison of Figs. 9 and 10 exhibits similarities in roof and maximum story drift patterns, but the values of maximum story drift being several times larger than the corresponding roof drift.

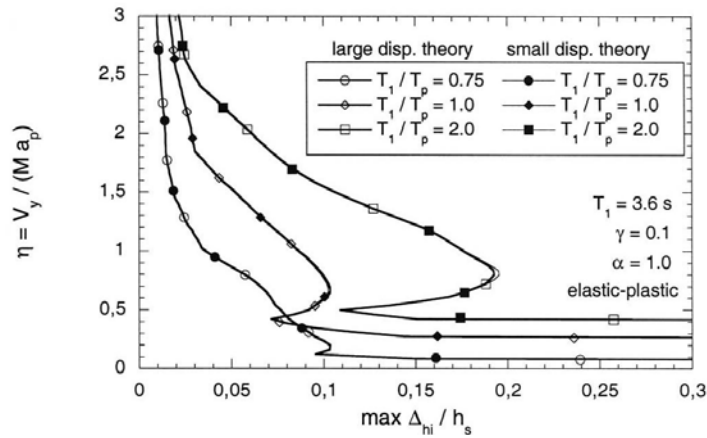


Figure 10. Relative strength parameter η versus maximum story drift for selected period ratios T_1 / T_p

Figure 11 shows horizontal displacement time histories recorded at the top of the first and second story, and at the roof for the 18-story generic frame exposed to a pulse P2. In all three cases the displacements are normalized by the height of a single story, h_s , and time t is normalized by $T_1 = 3.6$ s. An initial fundamental structural to pulse period ratio T_1 / T_p of 0.75 and a pulse intensity a_p / g of 0.114 ($\eta = 0.88$) are used. It can be seen that pulse P2 causes an oscillation with a negative and positive excursion, and leads to large residual displacements. Most of the inelastic deformation occurs in the second (positive) excursion, particularly in stories 1 and 2. The final displacement of the second story is already about 50 % of the roof displacement, indicating that most of the plastic deformation takes place in the lower stories. The displacement time histories show that large and small displacement theories lead to almost congruent results.

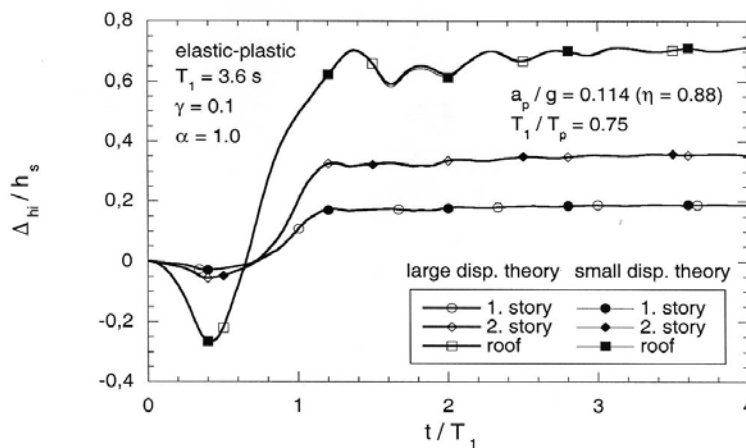


Figure 11. Horizontal displacement time histories at top of first and second stories and at roof level for a period ratio $T_1 / T_p = 0.75$, pulse intensity $a_p / g = 0.114$

4. EVALUATION OF THE MEMBER P-DELTA EFFECT

As already outlined in the introduction the "Updated Lagrangian" beam/column of the OpenSees computer program takes into account the so-called member P-delta effect, whereas in the formulation of the "Corotational" beam/column element only second order effects associated with displacements at the end of the members are considered (often referred to as structure P-delta effect). Member P-delta is related to the flexibility of the columns, and it addresses the additional moment of vertical loads at each section accounting for the displacement relative to the chord due to flexural deformations of the members. Commonly, in seismic structural engineering research and practice the latter effect is not included in analyses. As a byproduct of this investigation the question is addressed whether member P-delta gains on importance for the prediction of the seismic structural response at very large displacements.

Because the SDOF model considered in the foregoing studies consists of rigid columns without displacements relative to the chord it is not appropriate to evaluate member P-delta effects. A modified one-story portal frame according to Figure 12, composed of a rigid beam, two flexible columns, and two point masses is utilized for this study. When the structural yield strength is exceeded, plastic hinges develop in the columns at each corner. An elastic-perfectly plastic moment-chord rotation hysteresis rule is used to model inelastic behavior. Structural properties such as stiffness, mass and yield displacement are assigned in such a way that models of Figure 1 and Figure 12 lead to identical responses when the member P-delta effect is neglected.

The SDOF model of Figure 12 is dynamically excited in the same manner as the model of Figure 1. The results of this study conducted with the "Updated Lagrangian" beam/column element show that second-order effects associated with member deformation are negligible. Peak responses of models according to Figs 1 and 12 are almost identical and no difference is evident by inspection.

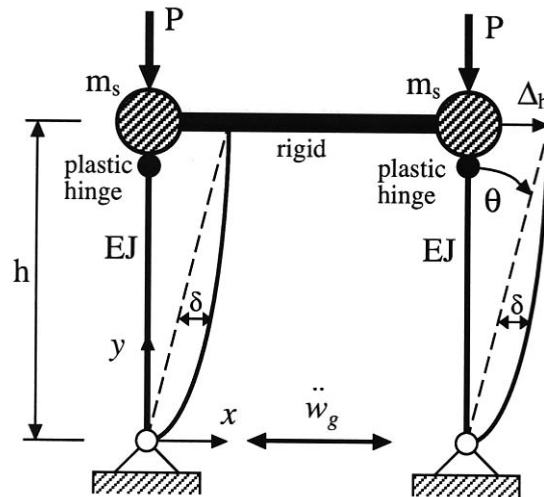


Figure 12. Idealized SDOF model of a single-story portal frame composed of flexible columns and rigid beam

The same conclusion can be drawn from the investigation of the previously discussed 18-story frame structure. Analyses were performed with both the "Updated Lagrangian" (member P-delta included) and the "Corotational" (without member P-delta) beam/column elements, both leading to almost identical response predictions.

5. CONCLUDING REMARKS

The results of this research shed light on elastic-plastic large displacement response characteristics of SDOF and MDOF frame structures subjected to pulse-type ground motions. As the main conclusions of this investigation it can be said that - within the assumptions made in this study - the influence of large displacement theory is of secondary importance for the response prediction of seismic excited elastic-plastic moment-resisting frame structures in its technical range of interest. Results of this study also show that second-order effects related to flexible deformations of members even at very large displacements are negligible for the structures investigated.

Acknowledgments: This research was carried out as part of a comprehensive effort at Stanford's John A. Blume Earthquake Engineering Center to assess seismic demands for frame structures. This effort is supported by the Pacific Earthquake Engineering Research (PEER) Center. The support of the first author in his research at Stanford University by a fellowship of the *Max Kade Foundation, Inc.* is gratefully acknowledged.

REFERENCES

1. Davies, M.A. and Moon, F.C., "Transition from soliton to chaotic motion following sudden excitation of a nonlinear structure". *Journal of Applied Mechanics* **63**(1996) 445-449.
2. Hjelmstad, K.D. and Williamson, E.B., "Dynamic stability of structural systems subjected to base excitation", *Engineering Structures*, **20**(1998) 425-432.
3. Williamson, E.B. and Hjelmstad, K.D., "Nonlinear dynamics of a harmonically excited inelastic pendulum". *Journal of Engineering Mechanics*, **127**(2001)52-57.
4. Adam, C. and Krawinkler, H. , "Large displacements of elastic-plastic frames exposed to pulse-type ground motion." In: Proc. of Response of Structures to Extreme Loading 2003 (XL2003), August 3-6, 2003, Toronto, Canada, CD-ROM paper, 8 pp. Elsevier 2003.
5. Vian, D. and Bruneau, M., "Experimental investigation of P-delta effects to collapse during earthquakes." *Technical Report MCEER-01-0001*, MCEER, University of Buffalo, Buffalo, 2002.
6. Alavi, B. and Krawinkler, H., "Effects of near-fault ground motions on frame structures." Report No. 138, The John A. Blume Earthquake Engineering Center, Stanford University, Stanford, 2001.
7. Medina R., "Seismic demands for nondeteriorating frame structures and their dependence on ground motions." *Ph.D. thesis*, Stanford University, Stanford, 2002.
8. Gupta, A., and Krawinkler, H. (2000). "Behavior of Ductile SMRFs at Various Seismic Hazard Levels," *Journal of Structural Engineering, ASCE*, No. 1, **126**(2000)98-107.

9. Aydinoglu, M. N., "Inelastic seismic response analysis based on story pushover curves including P-delta effects." *Report No. 2001/1*, KOERI, Department of Earthquake Engineering, Bogazici University, Istanbul, 2001.